

Shell evolution and its indication on the isospin dependence of the spin-orbit splitting

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The available experimental data on shell evolution indicate that the strength of the spin-orbit (SO) single-particle potential may be enhanced in neutron-rich nuclei. We observe that such a simple scheme destroys the Harmonic Oscillator (HO) magic numbers $N = 8$ and 20 and generates new SO magic numbers like $N = 6, 14, 16, 32$ and 34 . The traditional SO magic numbers like $N = 28$ and 50 and $N = 14$ seen in ^{22}O are eroded somehow in neutron-rich nuclei due to the sensitivity of larger- l orbitals to the depth of the central potential but they are more robust than the HO magic numbers. The $N = 82$ shell closure persists in neutron-rich nuclei while HO shell closures like $N = 40$ and 70 do not emerge. Both mechanisms contribute to enhancing the $N = 56$ and 90 gaps by splitting the $1d_{5/2}$ and $0g_{7/2}$ and the $0h_{9/2}$ and $1f_{7/2}$ orbitals.

Keywords: Shell evolution; Woods-Saxon potential; Spin-orbit coupling; Isospin dependence

One of the most important and challenging frontiers of nuclear structure physics is the study of nuclei at the limit of stability, especially neutron-rich nuclei with weakly bound neutrons. A topic of particular interest is the evolution of the shell structure in those nuclei. That is, the magic number may change dramatically depending on the N/Z ratio when we move towards the particle drip lines [1]. Such study is important not only due to the expected variation in properties of nuclei and the formation of island of inversion but also for the understanding of nuclear astrophysics as well as the nucleon-nucleon interaction. Nowadays it is rather commonly accepted that the $N = 8$ and 20 Harmonic Oscillator (HO) shell closures disappear in neutron-rich nuclei [1]. On the other hand, new magic numbers like $N = 14, 16$, and 32 may emerge. Shell model calculations suggest that $N = 34$ may also be a magic number in Ca isotopes, depending on the effective interactions used (see, e.g., Fig. 2 in Ref. [2]).

Among the intensive discussions on the mechanisms behind the shell evolution phenomena, we mention the tensor component of the two-body shell model interaction and the three-body interaction (see Refs. [1, 2] and references therein). In that scheme, the evolution of the shells are solely determined by the correlation between valence nucleons in the open shell. The shell evolution has also been analyzed from a self-consistent mean-field point of view without assuming an inert core (see, e.g., Ref. [1, 3]). It suggests that a significant part of the spin-orbit splitting may come from the two-body spin-orbit (SO) and tensor forces and three-body forces. The availability of experimental data in nuclei with large N/Z ratios may provide a ground to constrain the properties of different components of the interaction, such as the isovector channel of the SO interaction, which are not well defined but may be responsible for the shell evolution.

From a simple phenomenal point of view, the shell

structure is characterized by the presence of gaps in single-particle spectra. The HO mean field with SO coupling was the first successful model to predict correctly the traditional magic numbers [4]. The calculated shell structure may change if an isospin dependence of the SO coupling is taken into account. To illustrate this point, in Fig. 1 we evaluate the HO single-particle spectra by adding an isospin-dependent SO coupling of the form $\lambda(1 + \kappa_{SO} \frac{N-Z}{A}) \hbar \omega \mathbf{l} \cdot \mathbf{s}$ [5] to the HO potential. A negative κ_{SO} means that the SO coupling will gradually diminish when we go from $N = Z$ to neutron-rich nuclei, which indicates that SO shell closures like $N = 28$ and $N = 50$ will disappear and the spectra will be characterized by only HO magic numbers. However, this contradicts most available experimental information on the shell evolution even though there is indication that the $N = 28$ may have eroded in ^{42}Si [6–10]. On the other hand, a positive κ_{SO} would indicate that the SO coupling will be enhanced in neutron-rich nuclei. In such a scheme, the $N = 8$ and 20 HO magic numbers will disappear while SO shell closures like $N = 6, 14, 16, 32, 34$ and 56 will appear. It should be kept in mind, however, that the gaps at $N = 28$ and 50 will be enhanced in neutron-rich nuclei within this naive picture.

In reality, we may expect that the SO coupling will be reduced in neutron-rich drip-line nuclei with a diffusive surface since the SO interaction is peaked at the nuclear surface. One also has to consider that the single-particle orbitals may show different l dependence on the strength of the potential, which can lead to a systematic change of the shell structure [11]. Thus a more realistic description of the shell structure may be obtained with the Woods-Saxon potential which has a glorious history of success and is still one of the most suitable models in describing the nuclear single-particle structure. A variety of parameterizations of the Woods-Saxon potential exists (see, e.g., Refs. [5, 12–14] and Table II in Ref. [15]). In the “standard” one [5, 12], the strengths of the central and SO potentials are given as

$$V = V_0(1 + \frac{4\kappa}{A} \mathbf{t} \cdot \mathbf{T}_d), \quad (1)$$

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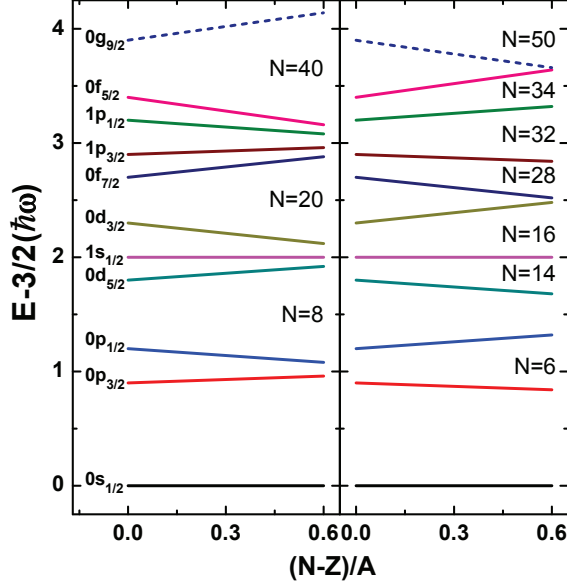


FIG. 1. (Color online) The evolution of the shell structure as a function of $(N - Z)/A$ with the HO potential plus SO coupling of the form $\lambda(1 + \kappa_{SO} \frac{N-Z}{A})\hbar\omega \mathbf{l} \cdot \mathbf{s}$. We take $\lambda = 0.2$ and $\kappa_{SO} = -1$ (left) and 1 (right). The $0g_{9/2}$ orbital is shifted upwards by $0.3\hbar\omega$ for a clearer presentation.

and

$$V_{SO} = \lambda V_0 \left(1 + \frac{4\kappa_{SO}}{A} \mathbf{t} \cdot \mathbf{T}_d\right), \quad (2)$$

where we have replaced the original $N - Z$ term with $4\mathbf{t} \cdot \mathbf{T}_d$ to get a consistent description of both protons and neutron orbitals. \mathbf{t} and \mathbf{T}_d denote the isospin quantum numbers of the last nucleon and the daughter nucleus, respectively. The total isospin of the system is $\mathbf{T} = \mathbf{t} + \mathbf{T}_{A-1}$. We have $4\mathbf{t} \cdot \mathbf{T}_{A-1} = -3$ for the $T = 0$ ground state of a $N = Z$ nucleus and

$$\begin{aligned} 4\mathbf{t} \cdot \mathbf{T}_{A-1} &= N - Z - 1 \text{ for neutron orbits} \\ &= -(N - Z + 3) \text{ for proton orbits} \end{aligned} \quad (3)$$

in $N > Z$ nuclei with $T = (N - Z)/2$ [15]. In Ref. [5], the isospin-dependent terms in Eqs. (1) and (2) are parameterized as

$$\kappa = \kappa_{SO} = -\frac{33}{51}, \quad (4)$$

where the SO potential depth is assumed to have the same isospin dependence as that of the central potential. This assumption is rather commonly adapted [14, 15]. The typical strength of κ is in the range $-0.6 \sim -0.9$.

We have done a systematic calculations on the shell evolution with the standard Woods-Saxon parameters (see, also, Ref. [11]). It suggests that SO shell closures like $N = 28, 50$ and 82 will erode in neutron-rich nuclei but in a manner that may be too fast, as can be seen in

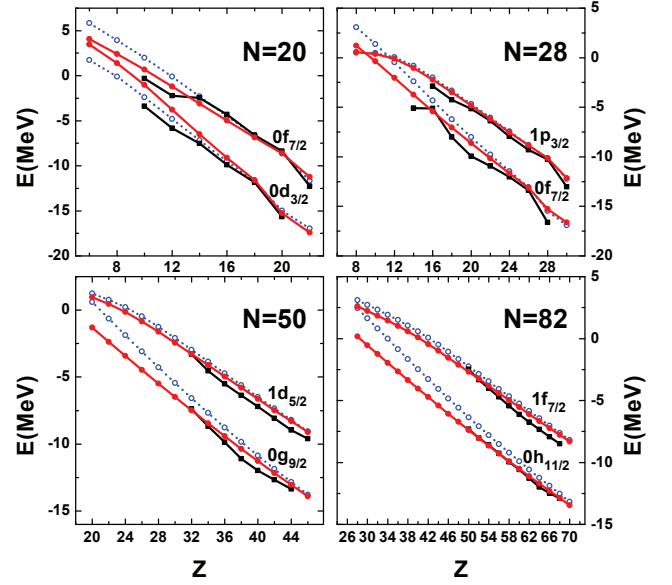


FIG. 2. (Color online) Evolution of the $N=20, 28, 50$ and 82 gaps as a function of proton number Z for calculations with the standard Woods-Saxon parameter and $\kappa_{SO} = \kappa$ (open circles) and $-\kappa$ (solid circles). The black squares denote the corresponding experimental data [16].

Fig. 2. Moreover, it cannot explain in a straight-forward way the disappearance of HO shell closures like $N = 8$ and 20 . The predicted $N = 40$ gap is also too strong. As we will show below, it seems that the problem may be fixed if we assume a strong positive κ_{SO} . Calculations with other parameters [14, 15] will lead to the same conclusion.

To get a qualitative idea on the role played by κ_{SO} , two kinds of calculations are done with the standard Woods-Saxon parameter [5] by taking $\kappa_{SO} = \kappa$ and $-\kappa$. The numerical code GAMOW is used [17]. Calculations on the evolution of the $N=20, 28, 50$ and 82 magic numbers are plotted in Fig. 2. The $N = 20$ shell closure is expected to disappear in neutron-rich nuclei like ^{32}Mg [1] (see, also, Ref. [18]). In calculations with the standard parameter, however, the gap in nucleus like ^{28}O is as large as 4.2 MeV. The $N = 20$ shell persists even if one takes $\kappa_{SO} = 0$, as shown in Ref. [15]. If one takes $\kappa_{SO} = -\kappa$, however, the $N = 20$ gap in ^{28}O would reduce to only 1.2 MeV.

The situation around $N = 28$ is somewhat complicated [6]. The $N = 28$ will be reduced in neutron-rich nuclei as expected from the standard Woods-Saxon calculations [11]. But the reduction of this shell gap will be significantly retarded if a positive κ_{SO} is assumed, as can be seen in Fig. 2. Similarly, the $N = 82$ shell closure may be reduced in very neutron-rich nuclei like ^{110}Ni since the large l orbital $0h_{11/2}$ will lose energy faster than other smaller- l orbitals when the potential becomes shallower. Indeed, the distance between the $0h_{11/2}$ and

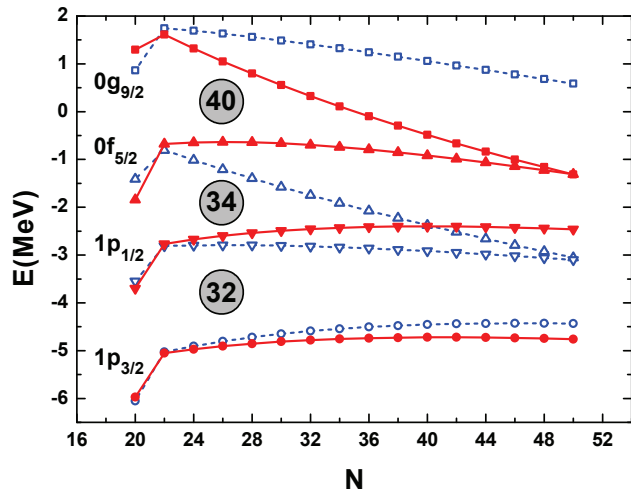


FIG. 3. (Color online) Evolution of the single-particle energies of the $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$ and $0g_{9/2}$ orbitals in Ca isotopes as a function of neutron number N for calculations with the standard Woods-Saxon parameter and $k_{SO} = k$ (open symbols) and $-k$ (solid symbols). In former calculations, the $N = 40$ shell gap is significantly enhanced in neutron-rich isotopes while the $N = 34$ sub-shell gap is eroded.

$1f_{7/2}$ orbitals decrease from 4.1 MeV in ^{132}Sn to 0.65 MeV in ^{110}Ni in calculations with the standard parameter. The gap is increased to 2.5 MeV if we simply take $\kappa_{SO} = -\kappa$. It should be mentioned that experimental information in Ni isotopes is known only up to $N = 50$. The $1f_{7/2}$ orbital in ^{110}Ni is predicted to be unbound in both calculations.

The sign of κ_{SO} can have significant influence on the relative strength of the $N = 40$ and 50 shell gaps in neutron-rich nuclei. As examples, in Fig. 3 we plot the single-particle spectra of Ca isotopes calculated with the standard parameter. The $N = 40$ shell gap in ^{60}Ca is predicted to be as large as 4.4 MeV in calculations with the standard parameter. While in the unbound nucleus ^{70}Ca the $N = 50$ gap is only 0.67 MeV. The $N = 50$ shell gap may disappear if the $0g_{9/2}$ orbital lose too much energy due to the shallowing of the central potential [11]. On the other hand, the gaps are calculated to be 0.43 MeV and 2.3 MeV, respectively, if we take $k_{SO} = -k$, which means that a positive k_{SO} will restore the $N = 50$ magic number by “destroying” the $N = 40$ HO shell closure. There is no experimental indication that $N = 40$ will emerge as a new magic number in neutron-rich nuclei [19, 20].

The $N = 32$ gap between orbitals $p_{3/2}$ and $p_{1/2}$ is not much affected by the κ_{SO} term. The $N = 32$ gap in ^{52}Ca is calculated to be 1.8 MeV with the standard parameter. It is increased to 2.3 MeV if we take $k_{SO} = -k$. The difference between $1p_{1/2}$ and $0f_{5/2}$ is only 0.92 MeV in ^{54}Ca . The $N = 34$ gap increased to 1.7 MeV if we take $k_{SO} = -k$.

Recent experiments suggest that ^{22}O and ^{24}O should be doubly magic nuclei [1] (see, also, Refs. [21–23] for recent results). ^{24}O is the heaviest bound oxygen isotope that has been observed so far. The $N = 16$ gap between the neutron $1s_{1/2}$ and $0d_{3/2}$ states are measured to be 4.86 ± 0.13 MeV in Ref. [21]. The values given by calculations with $\kappa_{SO} = \kappa$ and $-\kappa$ are 3.3 MeV and 4.5 MeV, respectively. The calculated $N = 14$ gap in ^{20}O increases from 1.8 MeV to 2.7 MeV if one takes $\kappa_{SO} = -\kappa$.

The $N = 14$ gap disappear in C and N isotopes with nearly degenerate $0d_{5/2}$ and $1s_{1/2}$ orbitals. This is easily understood since the $0d_{5/2}$ orbital loses its energy faster when going towards the dripline, resulting a nearly-degenerate $0d_{5/2}$ and $1s_{1/2}$ [11]. From a shell model point of view, where the above-mentioned mechanism is missing, this fact is related to the complicated interplay between the isovector and isoscalar two-body interactions [24].

A possible different form of isospin dependence in the SO potential than that of the central potential has been the subject of several studies [13, 15, 25]. Ref. [15] assumed that $\kappa_{SO} = 0$. The study of Ref. [13] showed that $\kappa_{SO} \sim 0.2$ to 0.7 , which is opposite to κ , can also explain the single-particle spectra in neutron-rich nuclei ^{132}Sn and ^{208}Pb . They suggest that such a opposite value is consistent with the two-body SO interaction as well as the Walecka and Skyrme-Hartree-Fock calculations. The study of Ref. [3] showed that the isoscalar SO coupling in the Skyrme force is reduced while the tensor coupling is strongly attractive, which may also indicate that the SO splitting can be enhanced in neutron-rich nuclei. However, it should be mentioned that the spectra of heavy stable nuclei are quite insensitive to the sign of κ_{SO} . As a result, it is not determined by a normal global fitting procedure [25].

The effect of the κ_{SO} term may be partly swallowed by other Woods-Saxon parameters, in particular the SO radius parameter r_{SO} in the Woods-Saxon potential. r_{SO} was taken as a free parameter in several calculations [14, 15], which tends to take values smaller than that of the central potential, i.e., r_0 . To explore this point further, we re-fitted the Woods-Saxon parameters under the restriction $\kappa_{SO} = -\kappa$. The parameters of the Woods-Saxon potential are adjusted to single-particle and single-hole states around the doubly-magic nuclei ^{16}O , $^{40,48}\text{Ca}$, ^{56}Ni , ^{100}Sn , ^{132}Sn and ^{208}Pb , as listed in Refs. [13, 15]. We use the same fitting procedure as employed in Ref. [26]. The fitting to nuclear single-particle energies usually favors larger values for the radius parameter r_0 , which may lead to a bad description of nuclear charge radii and moments of inertia [27]. In this work we restrict that $r_0 < 1.3$ fm. The results thus obtained are presented in Table I. Calculations on the evolution of the $N = 20, 28, 50$ and 82 magic numbers are plotted in Fig. 4, which show similar trends to those in Fig. 2.

In Fig. 5 we evaluate the influence of the isospin dependence in the SO coupling on the proton single-particle spectra in neutron-rich nuclei. As seen from Eq. (3), a

TABLE I. Woods-Saxon potential parameters obtained by fitting to single-particle and single-hole states around doubly-magic nuclei with the restriction $\kappa_{SO} = -\kappa$ and comparison with some existing parameters.

	V_0 (MeV)	r_0 (fm)	r_{SO} (fm)	a, a_{SO} (fm)	λ	κ
	50.92	1.285	1.146	0.691	24.07	0.644 $\kappa_{SO} = -\kappa$
Refs. [5, 12]	51	1.27	1.27	0.67	32.13	0.647 $\kappa_{SO} = \kappa$
Ref. [14]	49.6	1.347(n)/1.275(p)	1.31(n)/1.32(p)	0.7	35(n)/36(p)	0.86 $\kappa_{SO} = \kappa$
Ref. [15]	52.06	1.260	1.16	0.662	24.1	0.639 $\kappa_{SO} = 0$

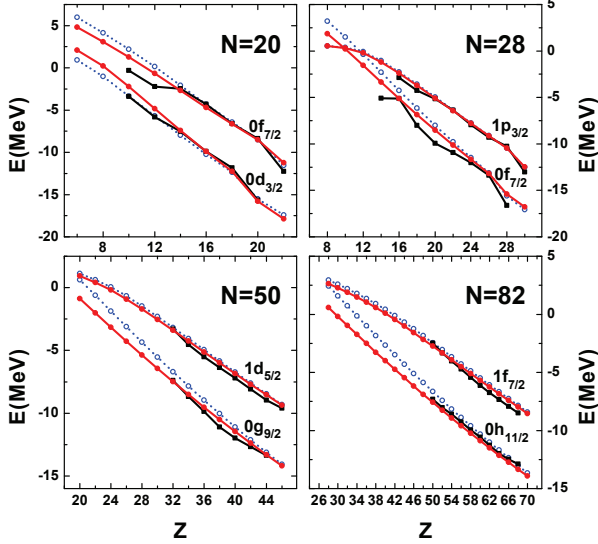


FIG. 4. (Color online) Same as Fig. 2 but for calculations with the re-fitted parameter and $\kappa_{SO} = \kappa$ (open symbols) and $\kappa_{SO} = -\kappa$ (solid symbols).

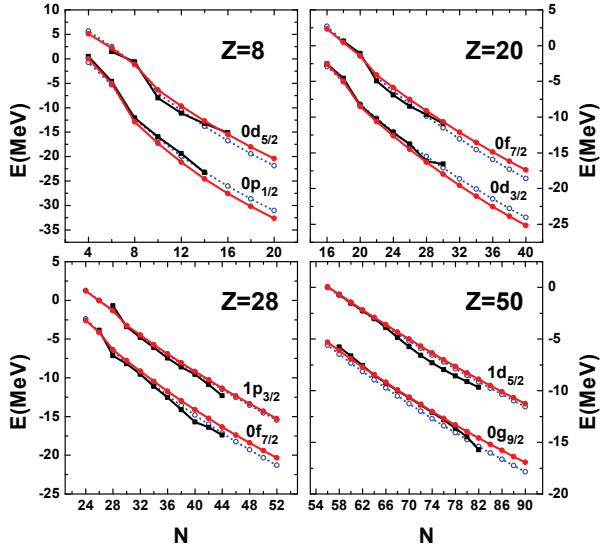


FIG. 5. (Color online) Same as Fig. 4 but for calculations on the $Z = 8, 20, 28$ and 50 proton shell gaps as a function of neutron number N in neutron-rich nuclei. The open and solid symbols stand for calculations with $\kappa_{SO} = \kappa$ and $\kappa_{SO} = -\kappa$, respectively.

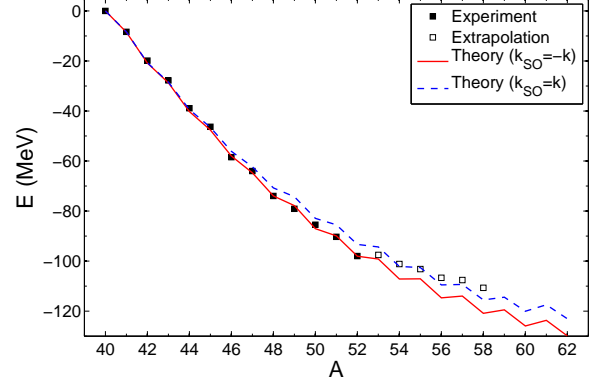


FIG. 6. (Color online) Experimental [34, 37] and calculated ground-state energies of Ca isotopes, relative to that of ^{40}Ca , as a function of mass number A .

positive κ_{SO} would suggest that the proton SO splitting is reduced in neutron-rich nuclei. Ref. [28] does show that the splitting between the proton shells $0g_{7/2}$ and $0h_{11/2}$ increases with neutron excess in neutron-rich Sb isotopes. As already shown in Ref. [29], this fact can be reproduced if we take a positive value for κ_{SO} . Whereas it is related the effect of the two-body tensor force within the Skyrme-Hartree-Fock approach [30]. It is difficult to conclude on the situation in light neutron-rich nuclei since experimental results are still inadequate. However, shell-model calculations tend to suggest that the splittings between SO partners like $0p_{3/2}$ and $0p_{1/2}$ and $0f_{7/2}$ and $0f_{5/2}$ may reduce with neutron excess [24, 31]. This is consistent with the binding energy systematics in Ref. [1] which shows that the $Z = 8$ and 20 gaps increase in neutron-rich nuclei. On the other hand, the $Z = 28$ gap may decrease [1, 32]. All these facts are consistent with our assumption that κ_{SO} is positive.

In this work we concentrated on the structure of light nuclei. But as shown in Ref. [13], the choice of a positive κ_{SO} is consistent with the single-particle spectra in ^{132}Sn and ^{208}Pb . Indeed, calculations with the re-fitted parameter in Table I can describe these states equally well in comparison with those with the potentials given in Refs. [14, 15].

The single-particle spectra may be influenced by correlation effects including deformation, particle vibration and pairing [27]. To explore the influence of pairing correlation, the single-particle plus pairing Hamiltonian is

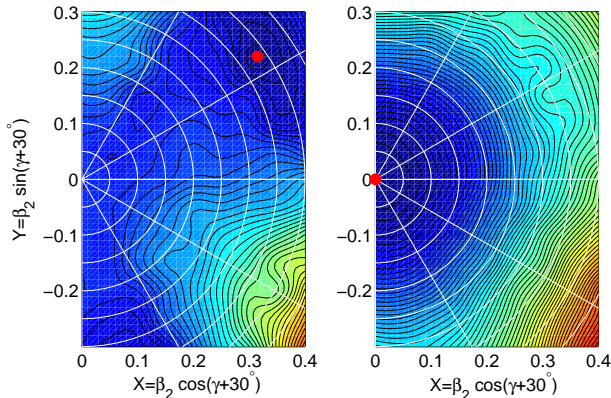


FIG. 7. (Color online) Potential energy surface (PES) calculations for ^{32}Mg ground state with the re-fitted Woods-Saxon parameter and $\kappa_{SO} = -\kappa$ (left) and κ (right). For each (β_2, γ) point the energy is minimized with respect to the β_4 deformation. The interval between neighboring contours is 0.1 MeV.

solved exactly with a Lanczos approach [33]. We assume that the ground states of even-even nuclei are all paired with seniority zero whereas those of odd- A nuclei are assumed to be of seniority one. We take the semi-magic Ca isotopes as examples, which have been studied recently both experimentally and theoretically [2, 34–36]. Calculations are done within a model space containing the $0f_{7/2}$, $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$, $0g_{9/2}$ and $1d_{5/2}$ neutron orbitals by assuming ^{40}Ca as the core. To minimize the number of free parameters, a simple constant pairing strength, $G = 1.795$ MeV, is employed in all calculations. The ground state energies thus calculated are presented in Fig. 6 together with experimental data. Calculations with $\kappa_{SO} = -\kappa$ predict larger binding energies for neutron-rich nuclei $^{51-58}\text{Ca}$ than the extrapolation values given in Ref. [37]. This is consistent with Ref. [34] where additional binding is measured for nuclei $^{51,52}\text{Ca}$. The increased binding in Ca isotopes may indicate a significant subshell gap at $N = 32$ [34]. There is a kink in the systematics of calculated two-neutron separation energies at $N = 34$, which suggests that $N = 34$ may also be a subshell. However, the $1p_{1/2}$ particle and $1p_{3/2}$ hole states are calculated to be nearly degenerate in the nucleus ^{53}Ca even though there is a noticeable gap in the calculated single-particle spectrum (c.f., Fig. 3).

To analyze the influence of nuclear deformation on the shell evolution, we evaluate the potential energy surfaces of the nucleus ^{32}Mg which has been intensively discussed recently [38–42]. Relativistic and non-relativistic mean-field as well as Woods-Saxon calculations result in a spherical shape for the ground state of this nucleus [43–45]. This problem may be related to the fact that the predicted $N = 20$ gap is rather large [46]. Whereas the observed large $B(E2)$ value for ^{32}Mg indicates that the nucleus is strongly deformed with $\beta_2 \sim 0.5$ [47]. Our cal-

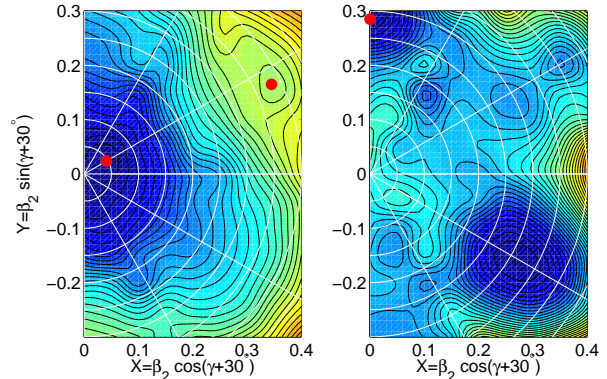


FIG. 8. (Color online) PES for the ground states in nuclei ^{34}Si (left) and ^{42}Si (right).

culations with the re-fitted parameter are plotted in Fig. 7. The minimum is around $\beta_2 = 0.38$ and $\gamma = 4.8^\circ$ for calculations with $\kappa_{SO} = -\kappa$. It should also be mentioned that the shape of ^{32}Mg is rather soft against β_2 deformation. The nucleus is calculated to be a rigid sphere if we take $\kappa_{SO} = \kappa$. Calculations with other parameters [14] lead to a similar conclusion.

In a recent paper the second 0^+ state in ^{34}Si was observed, which shows a large deformation parameter of $\beta_2 = 0.29$ [48]. The ground state of this nucleus is calculated to be spherical with a coexisting shallow deformed minimum (or more exactly a shoulder). The calculated deformation of the second minimum is around $\beta_2 = 0.38$. This second minimum in ^{34}Si disappears if we took $\kappa_{SO} = \kappa$. The nucleus ^{42}Si is calculated to be of oblate shape. This agrees with the shell model calculations with tensor force shown in Ref. [8] and the relativistic mean-field calculations in Ref. [49]. The ground state in the $N = 28$ isotone ^{44}Si is calculated to be of oblate shape with $\beta_2 = 0.27$ but the minimum is much shallower than that in ^{42}Si . It is expected that in this nucleus both deformed and spherical configurations coexist and mix weakly with each other [10].

It is expected that the quadrupole collectivity increases in neutron-rich nuclei around $N = 40$ [50–53]. This is supported by our calculations with $\kappa_{SO} = -\kappa$, as can be seen in Fig. 9. The calculated nuclear deformation in ^{64}Cr is close to that given by the five-dimensional quadrupole collective Hamiltonian calculation in Ref. [53]. Such an enhanced collectivity also indicates that $N = 40$ do not emerge as a magic number in neutron-rich nuclei.

In summary, we analyzed the shell structure of neutron-rich nuclei from a simple phenomenological single-particle point of view. If the SO splitting is relatively enhanced (i.e., with a strong positive value of κ_{SO}) in neutron-rich nuclei, both HO and Woods-Saxon calcu-

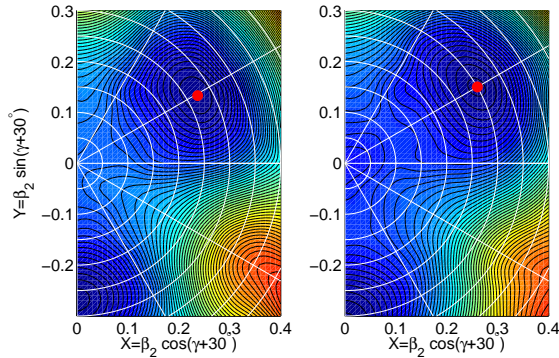


FIG. 9. (Color online) PES for $N = 40$ isotones ^{64}Cr (left) and ^{66}Fe (right). These nuclei are both calculated to be rigid spheres if we took $\kappa_{SO} = \kappa$.

lations show that it will destroy the HO magic numbers $N = 8$ and 20 and generate new SO magic numbers like $N = 6, 14, 16, 32$ and 34. The traditional SO magic numbers $N = 28$ and 50 and the magic number $N = 14$ will

be eroded somehow in neutron-rich nuclei due to the sensitivity of larger- l orbitals to the central potential depth. These SO shell closures are more robust than the HO magic numbers since their erosion is retarded by the SO splitting. In stable nuclei the $1d_{5/2}$ and $0g_{7/2}$ and the $0h_{9/2}$ and $1f_{7/2}$ orbitals are close to each other. These mechanisms both may split those shells, resulting in new shell closures like $N = 56$ and 90. This is in agreement with the Skyrme-Hartree-Fock calculations with tensor force shown in Ref. [3]. Such a simple scheme may give a quick estimation on the bulk properties of the single-particle spectra. It may also provide a convenient starting point for a variety of shell model calculations in the continuum (see, e.g., Ref. [54]) and to explore the effect of the pairing correlation and deformation which may influence the shell structure.

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